## Ant World

An 'Ant' is a cellular automaton which wanders over a grid of cells obeying certain rules.
Langton's original ant had 2 cell values ( $\mathbf{0}$ and $\mathbf{1}$ ), two directives ( $\mathbf{L}$ and $\mathbf{R}$ ) and a single state (A).
A natural extension of this ant (which we shall continue to call Langton's Ant) allows for multiple cell values and 2,4 or 6 directives - with the condition that the cell values are updated cyclically. In other words, if a cell has a value of 3 , it will be updated to value 4 (or 0 if there are only 4 cell values allowed). In this way, all that is needed to specify a Langton Ant is a string of directives. For example, the string LNRL would define an ant with 4 cell values $(0,1,2 \& 3)$. If this ant landed on a cell with a cell value of 2 , it would update it to 3 , turn right and move forward one space.

The four direction of a square grid are as follows:
$\mathbf{N} \quad$ No change of heading
L Turn left $90^{\circ}$
U Make a U turn
R Turn right $90^{\circ}$
and the six directions on a hexagonal grid are:
N No change of heading
L Turn left $60^{\circ}$
K Turn left $120^{\circ}$
U Make a U turn
Q Turn right $120^{\circ}$
R Turn right $120^{\circ}$
Whereas Langton's ants move one step at a time, Linton's ants move a number of steps equal to the current cell value plus one. i.e. if the ant moves onto a cell whose value is 3 , it will move in the direction specified 4 steps.

Langton's original ant has the defining string LR. After about 10,000 steps or random wanderings it builds a highway down to the SW corner. Linton's ant, with the same defining string does something completely different:


One of the most interesting single state ants is $\mathbf{L N}$ which builds a horizontal bar and counts in binary.

## $\square$ — $\boldsymbol{\sim}$ — $\square$

If the algorithm consists of pairs of left and right turns, the result is always bilaterally symmetric, Here are a couple of Langton examples using the string RRLL:


RRLL


LLRL

All the patterns exhibited here start from an empty grid. Patterns usually only emerge after a short period of chaos. It is perfectly possible that patterns can be 'seeded' with an initial state of cells. For example, it looks as if the ant on the left above could be persuaded to generate a blue redbordered square with a pair of white diagonals from an appropriate seed.

There are many single state ants which build a highway. Many are only 2 cells wide but some can be quite complex. The two ants illustrated below are rather rare. Both are 6 -valued ants. Each builds a highway and an ever expanding sail.


ULRRER


RNLRUR

One common behaviour is the building of a diamond with one or more diagonals. For example:


RULNUU


UNRRRN

Slightly less common behaviour is the building of a square. For example:


LLRLLL


LRLLLL

Finally, here are two rather unique examples:


Interesting patterns are more often produced using a hexagonal grid. The most common behaviour is a hexagonal spiral. Here are several examples:


LKK


## LKUK



URR


URURR

Here are two rather different and unusual patterns.


RKRUK


LURK

Here are two patterns generated by Linton's ant.


LUL


LRUUN

## 2-state Square Ants

Whereas Langton's and Linton's Ants only have one state, in general an ant may have any number of states. Its behaviour must be specified by a state table which tells what the ant must do given every possible combination of cell values and states. A typical entry is 0RC which means give the cell the value $\mathbf{0}$, turn Right and enter state $\mathbf{C}$.

The most interesting ants are those which build regular patterns. In the following examples the captions list first the algorithm used (Langton, Linton, Custom, Busy Beetle), then the kind of board (S2, S4, H2 or H6 where S stands for Square, H for Hexagonal and the number specifies the number of directives). The next two numbers specify the number of states and the number of allowed cell values. Finally the state table is listed in the following order $\mathbf{A 0}, \mathbf{A 1}, \ldots ; \mathbf{B 0}, \mathbf{B 1}, \ldots ; \mathbf{C 0}, \mathbf{C 1}, \ldots$ etc.

The simplest patterns are a solid square block or a solid diamond:


Custom Ant S2 22
1LA 1RB; OLA 1RA


Custom Ant S2 22
1LA 0RB ; ILA OLA
but there are many more interesting designs - e.g.


Custom Ant S2 22
1LA 0LB ; ORA 1RB


Custom Ant S2 22
OLB 1RA IRB 0LA


Custom Ant S2 22
ORB 1RB; ILB 0RA


Custom Ant S2 22
OLB ORA IRB OLA

If we allow multiple cell values, the patterns become more colourful but only the octagonal and kite-shaped ones are essentially different.


Custom Ant S2 23
2LA 0RA 1RB; 2LA 0RB 2LA


Custom Ant S2 23
2LA 1RB 2RB ; 2LA 0RB 1RA


Custom Ant S2 23
2RA 0LB ILB 0RA ORA 2LA


Custom Ant S2 24
2RA OLA ORB ORB; 1RA ORB OLB IRB


Custom Ant S2 24
ORB 3LA 1LA 3LB; ILB 1RA ORB 1LB


Custom Ant S2 24
2RB 0LA 3RA 3LB ; 2LB 3RA ILB 0RB


Custom Ant S2 24
2LA 1LA 1RB 3LA ; 2LA 2RB 3LA 2RA


Custom Ant S2 24
3LA 0RB 2LB 1LA ; 2RA 2RA 2RA 3LA


Custom Ant S2 24 3LB 3RB ORB 3LB;
3RB 1RA ILB 1RA


Custom Ant S2 26
4RB 2RB 4RB 5RA 3LA 4LB; 4RB 2LA 3RA 5LB ILB 0LA

If we allow four directives ( $\mathbf{N}, \mathbf{L}, \mathbf{U} \& \mathbf{R}$ ) then some new behaviour emerges. One builds a diagonal bar which counts in binary.


Custom Ant S4 22
OLB IRB; INA ORA
Another builds a spiral square but not in the way the you might think. It constantly returns to the origin and expands it design from the centre, not the edge!


Custom Ant S4 22
1RB 0LA ; ILB 0RA
and one which builds a kite:


Custom Ant S4 24
2RA IUB ILA 0LA ; 0NB 3NB 3NA 3RB
These are unusual too:


Custom Ant S4 24
OLB 3RB IRA 1RB; INA INB 3NB IUB


Custom Ant S4 24
3LB IUA 0LB 3UB ; 2NA 0UB 2NB ILA

## 2-state Hexagonal Bees

It will not come as a surprise to learn that on a hexagonal grid, most of the patterns generated are either hexagonal or triangular.


Custom Ant H2 22
1RB 1RB; 1LB 0RA


Custom Ant H2 22
ORB 0LB; 1RB 1LA


Custom Ant H2 23
2LA 2LB 1LB; ORA OLB 2RB


Custom Ant H2 23
2LB 1RA 1LA; ILB 2RA 1RA

No essentially new behaviour emerges when we increase the number of cell values or directives but I rather like these two patterns:


Custom Ant H6 22
IKB INA ; INA 0QA


Custom Ant H6 23
1RB 0NB 2QB; 2RB IQB 0UA

## Multiple state Square Ants

The most common pattern is a square mat. Here are some slightly different designs which I particularly like:


Custom Ant S2 32
ILB ORB ; ORC OLA ; ILC ORA


Custom Ant S2 32
OLB ORA ; ILB ORC ; ORA IRB


Custom Ant S2 32
OLC 0RA; IRB 0LA; IRB ILA


Custom Ant S2 32
1LB 1RC; 0LC 1RC; IRA 1RB

The following patterns were obtained using 4 states, 4 cell values and 2 directives. Each entry in the state table can therefore be one of $4 \times 4 \times 2=32$ possibilities. Since there are $4 \times 4=16$ entries in the table, the total number of possibilities is $32^{16}$ which is approximately the same as the number of molecules in a glass of water and vastly exceeds the number of stars on the observable universe. My search program examined over 100,000 random tables in the space of a few hours and selected over 200 patterns that did not either just shoot of to infinity or wander chaotically round the origin. Of these the great majority were either square of diamond mats. The following ones were the most interesting:


Custom Ant S2 44
2LB 2RC 2LC 1RA; 1LB 1RD 2RC 0RA; 2RD 2RD 3RD 3LD ; 2RC 1RB 1RA 3RC


Custom Ant S2 44
3RC 2LB 2RA 1RD ; 1RC 3LA 1LC 3LA; 3RD lLB 2LB 1LB ; 2LB 0RB 3RA 3LB


Custom Ant S2 44
3RD 3LB 2LA 1RD ; 3RA 3RC 3LB 3RD; 3LA 2RA 3LC 0RD ; 0RB 3RA 3RB 3LA


Custom Ant S2 44
2LA 2LC 3RC 1RA ; 2LC 0LA 1RB 3RC; ORA 0LA 1LC 0LD ; OLD 3LD 3RD 3LA

Here are a couple of 'sails':


Custom Ant S2 44
IRD 1LA 0LB 2LD ; ILA 3RA 3LB 2RB; OLB OLA OLA OLC; IRA 2LD 3RB ILC


Custom Ant S2 44
2RB 3LB 0RD 0LD ; ILC ILA 3LA 2LC; 3RA 3RB ORB 1LC; 1RB 3RB 3LD 3LB
and this is the most convoluted highway that I have ever seen!


## Custom Ant S2 44

2LB 2RD IRA ORB ; 2RC 3RA IRB 0LA;
IRD ORC OLD IRA ; 3LB IRB ILA ORD

## Busy Beetles

By extension of Turing's concept of a Busy Beaver, I define a Busy Beetle as an Ant with at least one entry with a 'halting state' (Z). A Champion Busy Beetle is defined as the Ant with a given number of cell values, states and directives, which beetles around for the greatest number of steps before halting (or, sometimes, visits the most number of cells before halting).

Here is a short table of Champion 1-state Busy Beetles:

| Values | Longevity | Example State Table |
| :--- | :--- | :--- |
| 2 | 5 | 1LA 0LZ |
| 3 | 9 | 1LA 2LA 0LZ |
| 4 | 25 | 1LA 2RA3RA 0LZ |
| 5 | 53 | 1LA 3RA 4RA 2LA 0LZ |
| 6 |  |  |

cell

The longest lived 2-state Busy Beetles which I have found (with the number of steps and the number of cells visited in brackets) are:


Busy Beetle (121, 41) S2 22
ILA 0LB ; OLZ 1RA


Busy Beetle $(485,96)$ S2 23
2LA 2RA 2LB; OLZ OLB ILA
and the longest 3-state Busy Beetles which I have found is:


Busy Beetle (878, 137) S2 32
ILB ORA ; ORC OLZ; ORA ORB
I shall conclude with a few examples of Busy Beetles on a hexagonal grid. (Should we call them Busy Bees?)


Busy Beetle $(188,63)$ H2 23
2LB 0RA 1LA ; 0RB 0LZ 2LA


Busy Beetle (488, 240) H2 23
ILA 2RA 0RB ; OLZ OLA OLA
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## List of Single State Ants

| Algorithm | Square Langton | Hexagonal Langton | Square Linton | Hexagonal Linton |
| :---: | :---: | :---: | :---: | :---: |
| N | Single highway (E) | Single highway (E) | Single highway (E) | Single highway (E) |
| L | Stable block (4) | Stable ring (6) | Stable block (4) | Stable ring (6) |
| K | --- | Stable triangle (3) | --- | Stable triangle (3) |
| U | Stable block (2) | Stable block (2) | Stable block (2) | Stable block (2) |
| LN | Binary bar | Chaos | Double highway (E) | Chaotic mesh |
| LL | Square block (4) | Stable ring (6) | Stable block (9) | Stable blob (53) |
| LK | --- | Chaos | --- | Chaos |
| LU | Square block (4) | Stable ring (6) | Chaos | Broad highway (NNW) |
| LQ | --- | Chaotic mesh | --- | Hexagonal mesh |
| LR | Classic highway | Bilateral doily | Diamond mesh | Hexagonal mesh |
| KN | --- | Double highway (E) | --- | Double highway (E) |
| KL | --- | Chaos | --- | Hexagonal mesh |
| KK | --- | Stable block (3) | --- | Double highway (NNW) |
| KU | --- | Stable block (3) | --- | Chaotic mesh |
| KQ | --- | Double highway (SSE) | --- | Hexagonal mat |
| KR | --- | Chaotic mesh | --- | Hexagonal open mesh |
| UN | Double highway (E) | Single highway (E) | Single highway (E) | Single highway (E) |
| UL | Stable block (4) | Stable ring (6) | Diamond mesh | Hexagonal open mesh |
| UK | --- | Stable triangle (3) | --- | Hexagonal mesh |
| UU | Stable block (2) | Stable block (2) | Single highway (W) | Single highway (W) |
| UQ | --- | Stable triangle (3) | --- | Hexagonal mesh |
| UR | Stable block (4) | Stable ring (6) | Diamond mesh | Hexagonal open mesh |
| LNN | Ternary bar | Chaos | Double highway (E) | Chaos |
| LNL | Chaos | Chaos | Double highway (E) | Chaos |
| LNK | --- | Chaos | --- | Chaos |
| LNU | Chaos | Chaos | Double highway (E) | Chaos |
| LNQ | --- | Chaos | --- | Broad highway (NNW) |
| LNR | Chaos | Chaos | Double highway (E) | Chaos |
| LLN | Binary bar | Chaos | Chaos | Chaos |


| LLL | Square block | Hexagonal ring | Triple highway (E) | Quadruple highway <br> (E) |
| :---: | :---: | :---: | :---: | :---: |
| LLK | --- | Chaos | --- | Chaos |
| LLU | Double highway (E) | Chaos | Diagonal highway (NNE) | Chaos |
| LLQ | --- | Triangular chaos | --- | Chaos |
| LLR | Quadruple highway (NW) | Chaos | Chaos | Chaos |
| LKN | --- | Chaos | --- | Chaos |
| LKL | --- | Chaos | --- | Broad highway (W) |
| LKK | --- | Hexagonal spiral | --- | Chaos |
| LKU | --- | Chaos | --- | Broad highway (W) |
| LKQ | --- | Chaos | --- | Chaos |
| LKR | --- | Chaos | --- | Quadruple highway <br> (E) |
| LUN | Vertical highway | Chaos | Chaos | Chaos |
| LUL | Horizontal highway | Chaotic mesh | Diamond mat | Hexagonal spiral mesh |
| LUK | --- | Chaos | --- | Chaos |
| LUU | Stable block (4) | Stable ring (6) | Chaos | Chaos |
| LUQ | --- | Chaos | --- | Hexagonal spiral mesh |
| LUR | Chaos | Chaos | Highway (NE) | Chaos |
| LQN | --- | Chaos | --- | Chaos |
| LQL | --- | Hexagonal maze | --- | Chaos |
| LQK | --- | Chaos | --- | Chaos |
| LQU | --- | Chaotic rings | --- | Hexagonal spiral mesh |
| LQQ | --- | Chaos | --- | Chaos |
| LQR | --- | Chaos | --- | Chaos |
| LRN | Horizontal highway | Chaos | Chaos | Chaos |
| LRL | Chaos | Hexagonal mesh | Broad highway (NNE) | Chaos |
| LRK | --- | Chaos | --- | Chaos |
| LRU | Stable block (10) | Stable block (24) | Diamond mesh | Chaos |
| LRQ | --- | Chaos | --- | Chaos |
| LRR | Chaos | Hexagonal mesh | Chaos | Chaos |
| KNN | --- | Chaos | --- | Double highway (E) |
| KNL | --- | Chaos | --- | Double highway (E) |
| KNK |  | Double highway |  | Double highway (E) |


| KNU | --- | Chaos | --- | Double highway (E) |
| :--- | :--- | :--- | :--- | :--- |
| KNQ | --- | Chaos | --- | Double highway (E) |
| KNR | --- | Chaos | --- | Double highway (E) |
| KLN | --- | Chaos | --- | Chaos |
| KLL | --- | Diagonal highway | --- | Chaos |
| KLK | --- | Chaos | Chaos | Chaos |
| KLU | --- | Chaos | Chaos |  |
| KLQ | --- | Chaos | Hexagonal mesh |  |
| KLR | --- | --- | Chaos |  |
| KKN | --- | Horizontal highway | --- | Double highway (E) |
| KKL | --- | Chaos | --- | Double highway <br> (NNW) |
| KKK | --- | Stable block (3) | --- | Double highway <br> (NNW) |
| KKU | --- | Stable block (7) | Double highway |  |
| (NNW) |  |  |  |  |


| KRR | --- | Chaos | --- | Chaos |
| :---: | :---: | :---: | :---: | :---: |
| UNN | Expanding bar | Expanding bar | Single highway (E) | Single highway (E) |
| UNL | Chaos | Chaos | Single highway (E) bar | Single highway (E) |
| UNK | --- | Chaos | --- | Single highway (E) |
| UNU | Horizontal highway | Horizontal highway | Single highway (E) bar | Single highway (E) |
| UNQ | --- | Chaos | --- | Single highway (E) |
| UNR | Chaos | Chaos | Single highway (E) bar | Single highway (E) |
| ULN | Chaos | Chaos | Triple highway (E) | Chaos |
| ULL | Horizontal highway | Hexagonal mesh | Triple highway (ENE) | Chaos |
| ULK | --- | Horizontal highway | --- | Chaos |
| ULU | Stable block (4) | Stable block (6) | Chaos | Chaos |
| ULQ | --- | Chaos | --- | Chaos |
| ULR | Stable block (10) | Stable block (24) | Chaos | Chaos |
| UKN | --- | Chaos | --- | Triple highway (E) |
| UKL | --- | Horizontal highway | --- | Chaos |
| UKK | --- | Chaos | --- | Chaos |
| UKU | --- | Stable block (3) | --- | Chaos |
| UKQ | --- | Stable block (6) | --- | Hexagonal mesh |
| UKR | --- | Chaos | --- | Chaos |
| UUN | Horizontal highway | Horizontal highway | Single highway (W) | Single highway (W) |
| UUL | Stable block (4) | Stable block (6) | Single highway (W) | Single highway (W) |
| UUK | --- | Stable block (3) | --- | Single highway (W) |
| UUU | Horizontal highway | Horizontal highway | Single highway (W) | Single highway (W) |
| UUQ | --- | Stable block (3) | --- | Single highway (W) |
| UUR | Stable block (4) | Stable block (6) | Single highway (W) | Single highway (W) |

